Accessing Isospin Symmetry Breaking Effects in Superallowed Beta Decays

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Precision Tests of SM through the first-row CKM unitarity

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

“Cabibbo Angle Anomaly (CAA)” ~3σ
Brief introduction of myself

Precision Tests of SM through the first-row CKM unitarity

\[ |V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \]

Kaon Decays

Single-Nucleon RC

Nuclear Structure Effects

Lattice QCD

Unified Framework of RC: Dispersion Relation; Effective Field Theory

Gamow-Teller Matrix Elements

\[ \lambda = g_A / g_V \]

Ab-Initio

Experiments
A new concept: Electroweak nuclear radii constrain the isospin breaking correction to $V_{ud}$

*Seng and Gorchtein, 2208.03037*
ISB corrections in superallowed nuclear beta decays

Superallowed beta decays of $T=1, J^p=0^+$ nuclei provide the best measurement of $V_{ud}$

Assuming isospin symmetry, the tree-level nuclear matrix element is completely fixed:

$$M^0_F = \langle f_0 | \hat{\tau}_+ | i_0 \rangle = \sqrt{2}$$

“bare Fermi matrix element”
Isospin symmetry breaking (ISB) correction alters the Fermi matrix element:

\[ |M_F|^2 = |\langle f | \hat{T}_+ | i \rangle|^2 = |M_F^0|^2 (1 - \delta_C) \]

Caused by isospin mixing of nuclear states, predominantly due to Coulomb repulsion between protons

\[ H = H_0 + V \quad \text{ISB} \]

\[ \delta_C = O(V^2) \quad 0.1\%~1\% \]

Crucial in obtaining a nucleus-independent \( F_t \)-value from the nucleus-dependent \( f_t \)-values:

\[ |V_{ud}|^2 F_t (1 + \Delta^V_R) = 2984.43 \text{ s} \]

\[ F_t = ft (1 + \delta'_R) (1 + \delta_{NS} - \delta_C) \]

Hardy and Towner, 2020 PRC
**ISB corrections in superallowed nuclear beta decays**

- Computing $\delta_C$: Classic problem over 6 decades! *MacDonald, 1958 Phys.Rev*

- Current input adopted in global analysis: Shell model + Woods-Saxon (WS) potential by Hardy and Towner

- Successful in aligning Ft values of different superallowed transitions

  *Hardy and Towner, 2020 PRC*

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(Selected results)

**Caveats:**

- Significant model dependence. Disagreement with Hartree-Fock, DFT, RPA...

- **Theory inconsistencies**, e.g. not using the correct *isospin operator* *Miller and Schwenk, 2008 PRC, 2009 PRC; Condren and Miller, 2201.10651*

- **Results solely from nuclear models, no direct experimental constraint!**

- Ab-initio calculations still in preliminary stages

  *Caurier et al., 2002 PRC; Martin et al., 2021 PRC*
Probing isospin mixing effects from electroweak radii

Vibrant experimental programs of the **neutron skin measurements** with **parity-violating elastic scattering (PVES)** (for EoS, nuclear astrophysics)

\[ S_n = R_n - R_p \]

\[ R_{p/n,\phi} = \sqrt{\frac{1}{X} \langle \phi \mid \sum_{i=1}^{A} r_i^2 \left( \frac{1}{2} + \hat{T}_z(i) \right) \mid \phi \rangle} \]

- Ideally, measuring the neutron skin of the N=Z state in the isotriplet provides a clean probe of ISB effect:
  \[ \langle 1, 0 \mid \hat{O}_0^1 \mid 1, 0 \rangle = 0 \]

- Unfortunately, the N=Z state is unstable, rendering fix-target scattering experiments impossible
Probing isospin mixing effects from electroweak radii

The Tz=+1 state is always the most stable!

However, at N≠Z, disentangling the ISB and symmetry energy contribution to the neutron skin is non-trivial.
Probing isospin mixing effects from electroweak radii

Strategy: Measure different radii across the isotriplet!

“Isovector monopole operator”:

\[
M^{(1)} = \sum_{i=1}^{A} r_i^2 \vec{T}(i)
\]

Let’s consider \((T_z)_i = 0, (T_z)_f = +1\).

Measurement (1): t-dependence in beta decay

Beta decay form factors:

\[
\langle f(p_f) | J_{W}^{\lambda \dagger}(0) | i(p_i) \rangle = f_+(t)(p_i + p_f)^\lambda + f_-(t)(p_i - p_f)^\lambda
\]

Recoil effects probe the t-dependence, give the off-diagonal matrix element of the isovector monopole operator:

\[
\bar{f}_+(t) = 1 - \frac{t}{6} \frac{\langle f | M^{(1)}_{+1} | i \rangle}{\sqrt{2M_F}} + \mathcal{O}(t^2)
\]

Existing recoil expt: TRIUMF, ISOLDE (CERN) etc. Future expt at FRIB?
Probing isospin mixing effects from electroweak radii

Strategy: Measure different radii across the isotriplet!

“Isovector monopole operator”:

\[
\mathcal{M}^{(1)} = \sum_{i=1}^{A} r_i^2 \hat{T}(i)
\]

Let’s consider \((T_z)_i = 0, (T_z)_f = +1\).

**Measurement (2): p/n distribution radius at \((T_z)_f = +1\)**

For stable daughter nucleus, fixed-target scattering can be performed to measure \(R_p\) and \(R_n\) respectively (deduced from charge and weak radii)

Can combine to get another matrix element of the isovector monopole operator:

\[
\langle f | M_0^{(1)} | f \rangle = \langle f | \sum_{i=1}^{A} r_i^2 \hat{T}_z(i) | f \rangle = \frac{N}{2} R_{n,f}^2 - \frac{Z}{2} R_{p,f}^2
\]

PREX, CREX (JLab), P2, MREX (Mainz)…
Probing isospin mixing effects from electroweak radii

Strategy: Measure different radii across the isotriplet!

“Isovector monopole operator”:

\[ M^{(1)} = \sum_{i=1}^{A} r_i^2 \vec{T}(i) \]

Let's consider \((Tz)_i = 0\), \((Tz)_f = +1\).

**Combined experimental observable**

If isospin symmetry is exact, the two matrix elements are equal and opposite:

\[ \langle f_0 | M^{(1)}_{+1} | i_0 \rangle = -\langle f_0 | M^{(1)}_{0} | f_0 \rangle \]

Therefore, the combined experimental observable:

\[ \Delta M_A^{(1)} \equiv \langle f | M^{(1)}_{+1} | i \rangle + \langle f | M^{(1)}_{0} | f \rangle \]

provides a clean probe of ISB. **Deviation from zero signifies isospin mixing**
Probing isospin mixing effects from electroweak radii

Strategy: Measure different radii across the isotriplet!

“Isovector monopole operator”:

\[
\vec{M}^{(1)} = \sum_{i=1}^{A} r_i^2 \vec{T}(i)
\]

Let’s consider \((Tz)_i = 0\), \((Tz)_f = +1\).

**Measurement (3): Charge radii across the isotriplet**

**Nuclear charge radii** are measurable for both stable and unstable nuclei (through atomic spectroscopy)

Assuming \(R_{ch} \approx R_p\), the following observable is also a clean probe of ISB:

\[
\Delta M_B^{(1)} = \frac{1}{2} \left( Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2 \right) - Z_0 R_{p,0}^2
\]

Possible future measurements: BECOLA at FRIB
Connection to the ISB correction to $M_F$

Leading source of isospin mixing:
Isovector Coulomb potential in a uniformly charged sphere

$$V_C^{(1)} = \frac{Ze^2}{8\pi R_C^3} \sum_i r_i^2 \hat{T}_z(i) + ...$$

$$= \frac{Ze^2}{8\pi R_C^3} M_0^{(1)} + ...$$

Damgaard, 1969 Nucl.Phys.A; Miller and Schwenk, 2008 PRC; Auerbach, 2009 PRC

$\Delta M^{(1)}_A \approx -\frac{8\pi R_C^3}{Ze^2} \left\{ \frac{1}{3} \sum_a \frac{|\langle a; 0| V_C^{(1)} |g; 1 \rangle|^2}{E_{a,0} - E_{g,1}} + \frac{1}{2} \sum_{a \neq g} \frac{|\langle a; 1| V_C^{(1)} |g; 1 \rangle|^2}{E_{a,1} - E_{g,1}} + \frac{7}{6} \sum_a \frac{|\langle a; 2| V_C^{(1)} |g; 1 \rangle|^2}{E_{a,2} - E_{g,1}} \right\}$

$\Delta M^{(1)}_B \approx -\frac{8\pi R_C^3}{Ze^2} \left\{ \frac{2}{3} \sum_a \frac{|\langle a; 0| V_C^{(1)} |g; 1 \rangle|^2}{E_{a,0} - E_{g,1}} - \sum_{a \neq g} \frac{|\langle a; 1| V_C^{(1)} |g; 1 \rangle|^2}{E_{a,1} - E_{g,1}} + \frac{1}{3} \sum_a \frac{|\langle a; 2| V_C^{(1)} |g; 1 \rangle|^2}{E_{a,2} - E_{g,1}} \right\}$

$\delta_C \approx \frac{1}{3} \sum_a \frac{|\langle a; 0| V_C^{(1)} |g; 1 \rangle|^2}{(E_{a,0} - E_{g,1})^2} + \frac{1}{2} \sum_{a \neq g} \frac{|\langle a; 1| V_C^{(1)} |g; 1 \rangle|^2}{(E_{a,1} - E_{g,1})^2} - \frac{5}{6} \sum_a \frac{|\langle a; 2| V_C^{(1)} |g; 1 \rangle|^2}{(E_{a,2} - E_{g,1})^2}$

They share identical reduced matrix elements in the $T=0,1,2$ channels!

Benefits to theorists: Methodologies capable to compute $\delta_C$ can also compute $\Delta M^{(1)}$; the latter can be directly compared to experiment!
Further modeling invoking \textit{isovector monopole dominance}:

\[
\sum_a \frac{|\langle a; T || V_C^{(1)} || g; 1 \rangle|^2}{(E_{a,T} - E_{g,1})^n} \rightarrow \frac{|\langle M; T || V_C^{(1)} || g; 1 \rangle|^2}{(E_{M,T} - E_{g,1})^n}
\]

Assuming degenerate reduced matrix elements:

\[
\langle M; T || V_C^{(1)} || g; 1 \rangle = \mu \quad \text{(correction \sim |N-Z|/A)}
\]

Energy splitting:

\[
E_{M,T} - E_{g,1} = \xi \omega [1 + (T^2 + T - 4)\kappa/2]
\]

Reduced matrix elements drop out, leading to a direct proportionality:

\[
\delta_C \approx -\frac{Z^2 e^2}{8\pi R_C^3} \frac{\kappa(4 - 13\kappa + 12\kappa^2 - \kappa^3)}{(\kappa^2 - 4\kappa + 2)(1 - 2\kappa)(1 - \kappa^2)} \frac{1}{\xi \omega} \Delta M_A^{(1)}
\]

\[
\approx -\frac{Z^2 e^2}{8\pi R_C^3} \frac{(4 - 13\kappa + 12\kappa^2 - \kappa^3)}{2\kappa(1 - 2\kappa)(1 - \kappa^2)} \frac{1}{\xi \omega} \Delta M_B^{(1)}
\]

Proportionality constants bearing residual model dependence
Targeted Experimental Precision

How precise should experiments be to start probing isospin mixing effects?

The simple proportionality relation \( \delta_C \) available on market provide useful guidance on the targeted experimental precision!

<table>
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<tr>
<th>Transitions</th>
<th>( \delta_C ) (%)</th>
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<th>( \frac{\Delta M_A^{(1)}}{AR^2/4} ) (%)</th>
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<td>0.329</td>
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<td>( ^{34}\text{Cl} \to ^{34}\text{S} )</td>
<td>0.613</td>
<td>0.75</td>
<td>0.57</td>
</tr>
<tr>
<td>( ^{38}\text{K} \to ^{38}\text{Ar} )</td>
<td>0.628</td>
<td>1.7</td>
<td>0.59</td>
</tr>
<tr>
<td>( ^{42}\text{Sc} \to ^{42}\text{Ca} )</td>
<td>0.690</td>
<td>0.77</td>
<td>0.42</td>
</tr>
<tr>
<td>( ^{46}\text{V} \to ^{46}\text{Ti} )</td>
<td>0.620</td>
<td>0.563</td>
<td>0.38</td>
</tr>
<tr>
<td>( ^{50}\text{Mn} \to ^{50}\text{Cr} )</td>
<td>0.660</td>
<td>0.476</td>
<td>0.35</td>
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<td>( ^{54}\text{Co} \to ^{54}\text{Fe} )</td>
<td>0.770</td>
<td>0.586</td>
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\( \delta_C \) from different models

\( \Delta M_A^{(1)} \) sensitivity enhanced by \( 1/\kappa \)

Expected size of \( \Delta M_A^{(1)} \)

Targeted relative precision of \( R_p^2, R_n^2 \) measurement
Targeted Experimental Precision

How precise should experiments be to start probing isospin mixing effects?

The simple proportionality relation + $\delta_C$ available on market provide useful guidance on the targeted experimental precision!

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$\Delta M_B^{(1)}$ sensitivity suppressed by $\kappa$, but partial results already exists
Anticipated Synergies

Differential superallowed decay rate (FRIB, ...)

Scrutinize the theory relation (nuclear models, ab-initio, ...)

Charge and weak radii measurement (JLab, Mainz, FRIB, ...)

EoS, nuclear astrophysics

Precision tests of SM; Search for BSM physics
Summary

• Isospin breaking correction $\delta_C$ is an important SM theory input for the test of first-row CKM unitarity. Current determination suffers from theoretical inconsistencies, model-dependence, and is short of direct experimental constraint.

• We propose a new set of experimental observables $\Delta M_A^{(1)}, \Delta M_B^{(1)}$ from the measurement of electroweak nuclear radii across the isotriplet, as a clean probe of isospin mixing effects.

• We show that $\Delta M_A^{(1)}, \Delta M_B^{(1)}$ probe the same physics as $\delta_C$, thus serves as a strong constraint to the latter. They also provide useful consistency checks to theory calculations.

• Existing models indicate that measurements of $R_p^2, R_n^2$ to ~1% may start to probe isospin mixing effects.

• The new idea may motivate synergies between theory and experimental communities in physics of rare isotopes, electron scattering and nuclear astrophysics.