# Accessing Isospin Symmetry Breaking Effects in Superallowed Beta Decays


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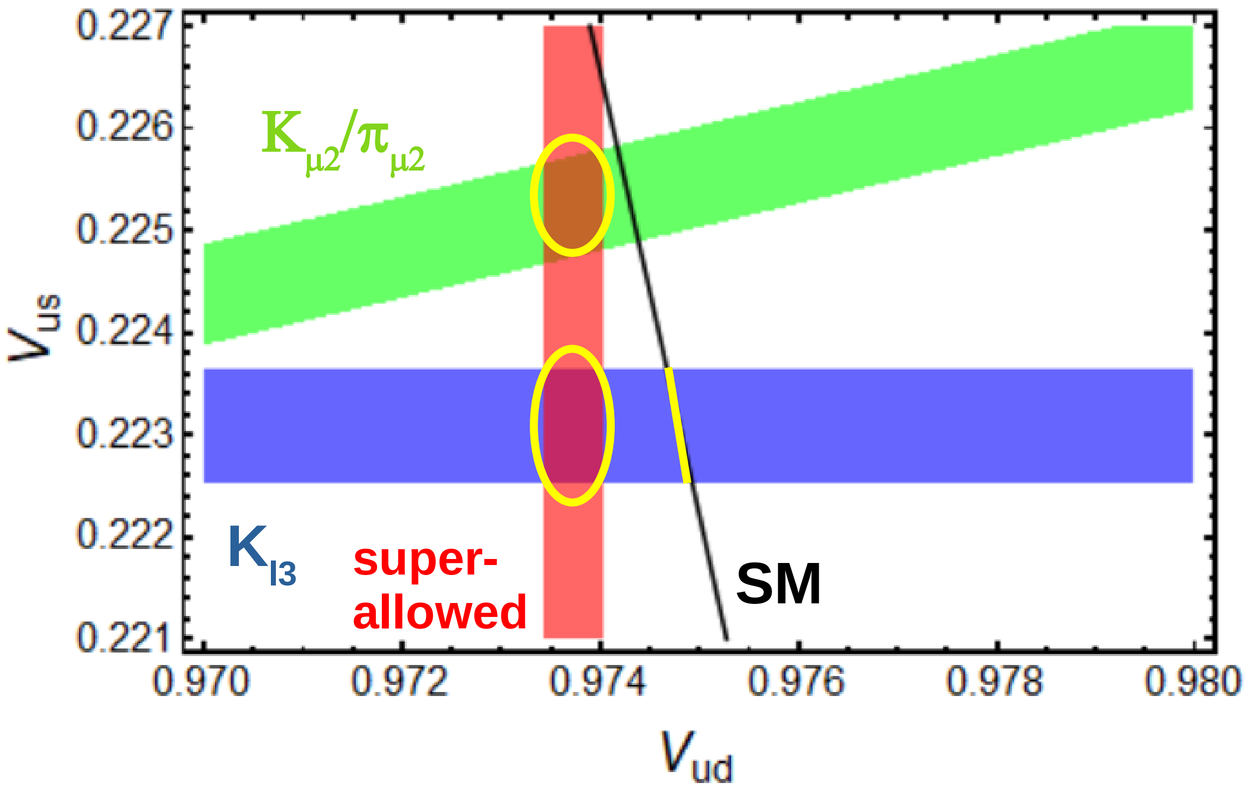
## Precision Tests of SM through the first-row CKM unitarity

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=1$$

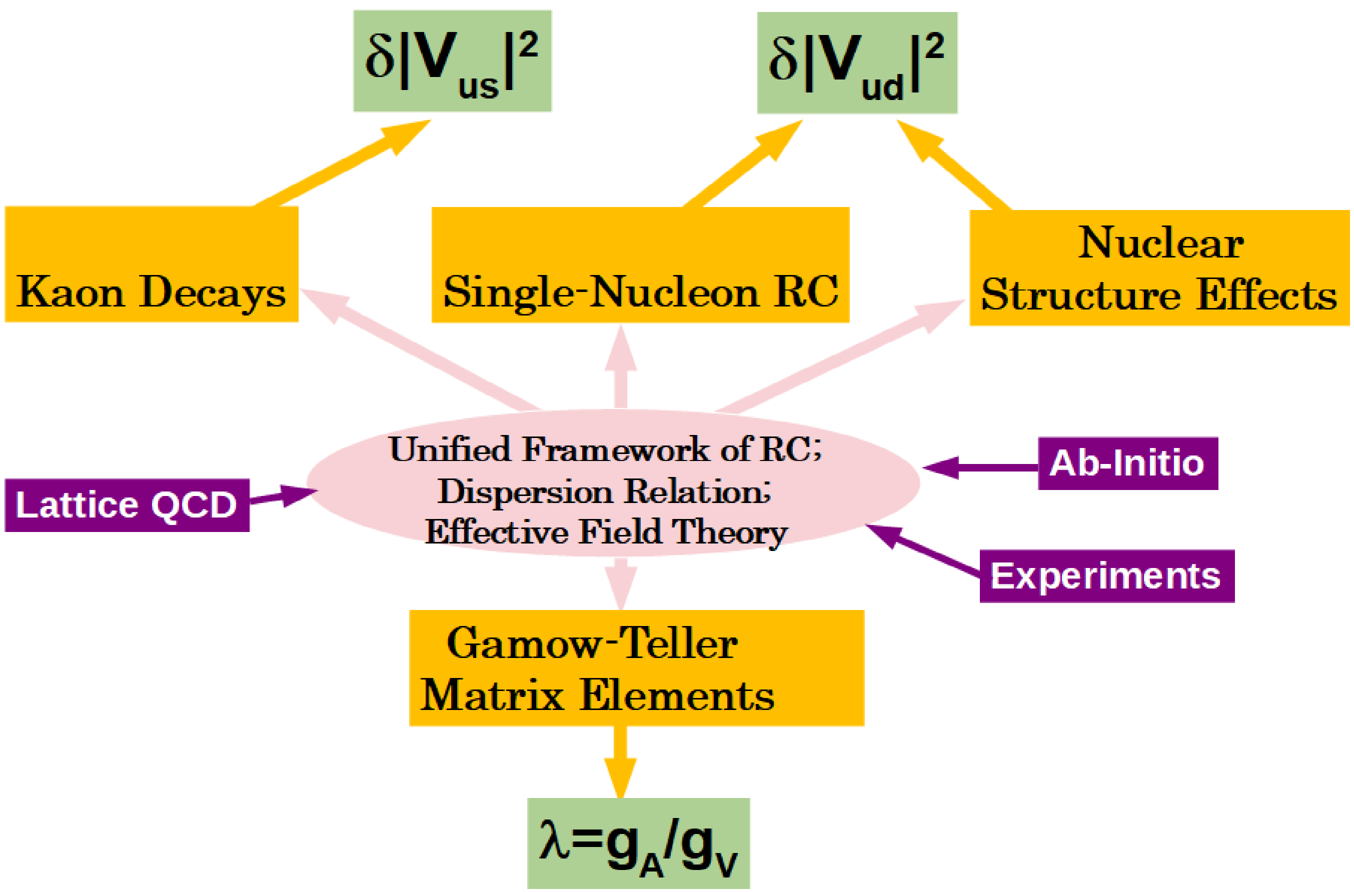



**"Cabibbo Angle Anomaly (CAA)" ~3σ**

# Brief introduction of myself

## Precision Tests of SM through the first-row CKM unitarity

$$|V_{ud}|^2+|V_{us}|^2+|V_{ub}|^2=1$$



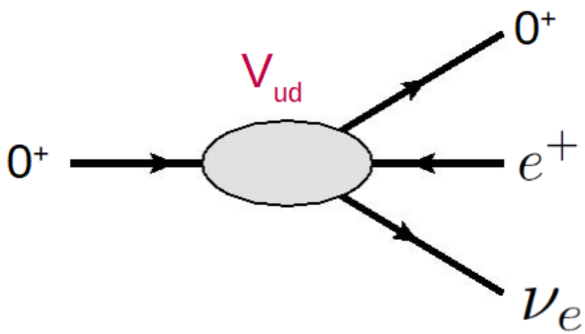
## A new concept:

# Electroweak nuclear radii constrain the isospin breaking correction to $V_{ud}$

*Seng and Gorchtein, 2208.03037*

# ISB corrections in superallowed nuclear beta decays

**Superallowed beta decays of T=1, $J^p=0^+$ nuclei** provide the **best measurement of $V_{ud}$**




$$|V_{ud}|^2 \mathcal{F}t\,(1+\Delta_R^V) = 2984.43\,\mathrm{s}$$

| | $\lvert V_{ud}\rvert$ |
|---|---|
| Superallowed nuclear decays ($0^+ \to 0^+$) | 0.97373(31) |
| Free $n$ decay | 0.97377(90) |
| Mirror nuclei decays | 0.9739(10) |
| Pion semileptonic decay ($\pi_{e3}$) | 0.9740(28) |

Assuming **isospin symmetry**, the tree-level nuclear matrix element is completely fixed:

$$M_F^0 \;=\; \langle f_0|\hat{\tau}_+|i_0\rangle = \sqrt{2}$$

"bare Fermi matrix element"

# ISB corrections in superallowed nuclear beta decays

Isospin symmetry breaking **(ISB)** correction alters the Fermi matrix element:

$$|M_F|^2 = |\langle f|\hat{\tau}_+|i\rangle|^2 = |M_F^0|^2(1-\delta_C)$$

Caused by **isospin mixing** of nuclear states, predominantly due to Coulomb repulsion between protons

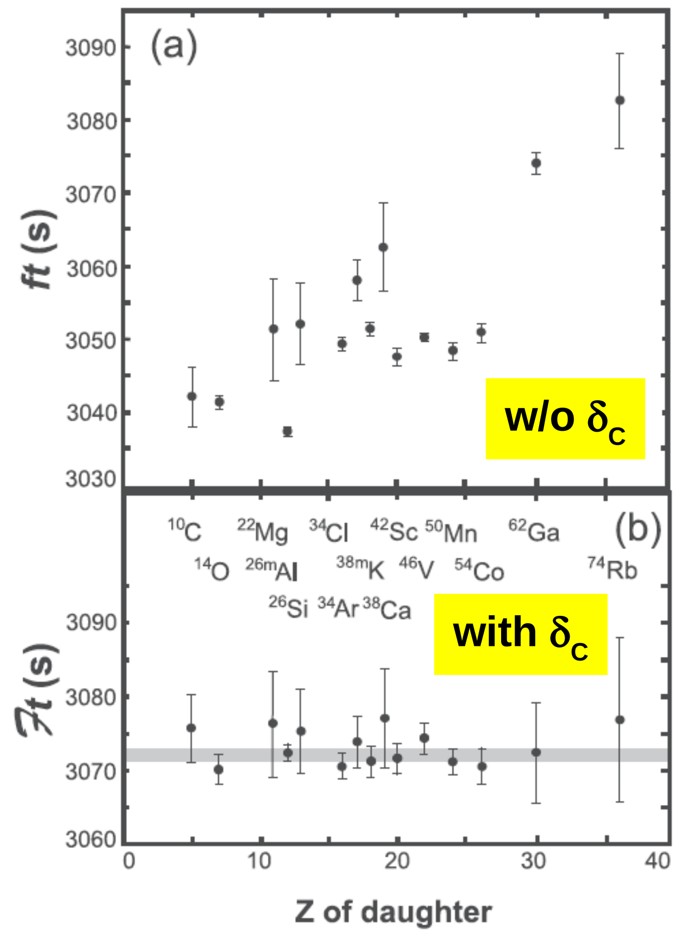
$$H = H_0 + V \quad \text{ISB}$$

$$\delta_C = \mathcal{O}(V^2) \quad 0.1\%\sim1\%$$

Crucial in obtaining a nucleus-independent Ft-value from the nucleus-dependent ft-values:

$$|V_{ud}|^2 \mathcal{F}t (1+\Delta_R^V) = 2984.43\,\mathrm{s}$$

$$\mathcal{F}t = ft\,(1+\delta'_R)\,(1+\delta_{NS}-\delta_C)$$




*Hardy and Towner, 2020 PRC*

# ISB corrections in superallowed nuclear beta decays

- Computing $\delta_c$: Classic problem over ***6 decades***! *MacDonald, 1958 Phys.Rev*
- Current input adopted in global analysis: Shell model + Woods-Saxon (WS) potential by Hardy and Towner
- Successful in aligning Ft values of different superallowed transitions

*Hardy and Towner, 2020 PRC*

| Transitions | $\delta_C$ (%) | | | | |
|---|---|---|---|---|---|
| | WS | DFT | HF | RPA | Micro |
| $^{26m}Al \to ^{26}Mg$ | 0.310 | 0.329 | 0.30 | 0.139 | 0.08 |
| $^{34}Cl \to ^{34}S$ | 0.613 | 0.75 | 0.57 | 0.234 | 0.13 |
| $^{38m}K \to ^{38}Ar$ | 0.628 | 1.7 | 0.59 | 0.278 | 0.15 |
| $^{42}Sc \to ^{42}Ca$ | 0.690 | 0.77 | 0.42 | 0.333 | 0.18 |
| $^{46}V \to ^{46}Ti$ | 0.620 | 0.563 | 0.38 | / | 0.21 |
| $^{50}Mn \to ^{50}Cr$ | 0.660 | 0.476 | 0.35 | / | 0.24 |
| $^{54}Co \to ^{54}Fe$ | 0.770 | 0.586 | 0.44 | 0.319 | 0.28 |

(Selected results)

**Caveats:**

- Significant model dependence. Disagreement with Hartree-Fock, DFT, RPA...
- Theory inconsistencies, e.g. not using the correct isospin operator *Miller and Schwenk, 2008 PRC, 2009 PRC; Condren and Miller, 2201.10651*
- **Results solely from nuclear models, no direct experimental constraint!**
- Ab-initio calculations still in preliminary stages

*Caurier et al., 2002 PRC; Martin et al., 2021 PRC*

# Probing isospin mixing effects from electroweak radii

Vibrant experimental programs of the **neutron skin measurements** with **parity-violating elastic scattering (PVES)** (for EoS, nuclear astrophysics)

$$S_n = R_n - R_p$$

*PREX, CREX, P2, MREX...*

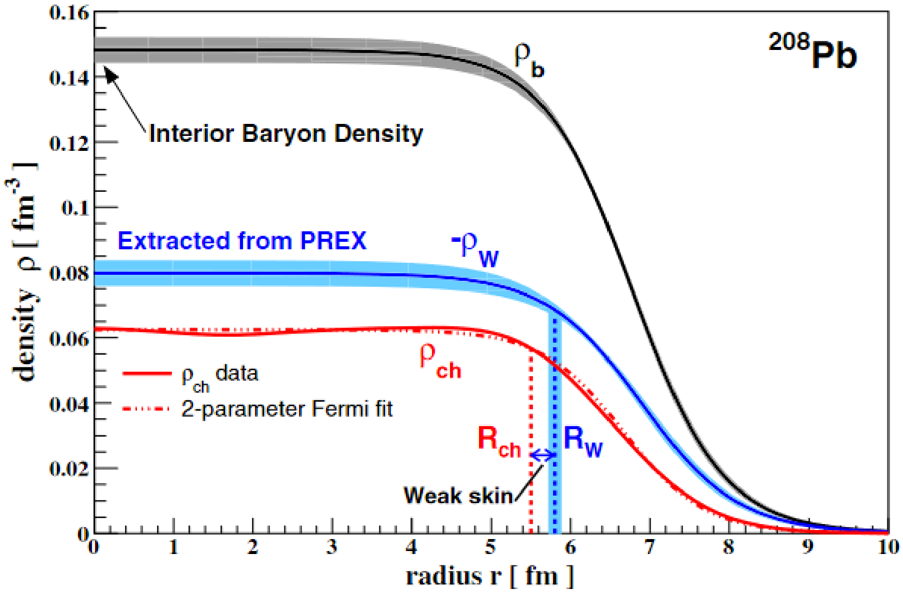
$$R_{p/n,\phi} = \sqrt{\frac{1}{X}\langle\phi|\sum_{i=1}^{A} r_i^2 \left(\frac{1}{2} \mp \hat{T}_z(i)\right)|\phi\rangle}$$

X=Z or N

- Ideally, measuring the neutron skin of the N=Z state in the isotriplet provides a clean probe of ISB effect:

$$\langle 1,0|\hat{O}_0^1|1,0\rangle = 0$$

- Unfortunately, the N=Z state is unstable, rendering fix-target scattering experiments impossible




*PREX Collaboration, 2021 PRL*

# Probing isospin mixing effects from electroweak radii

The Tz=+1 state is always the most stable!

| Tz = -1 (Parent) | 0 (daughter) | +1 | |
|---|---|---|---|
| $^{10}_{6}$C | $^{10}_{5}$B (ex) | $^{10}_{4}$Be | $10^6$ yrs |
| $^{14}_{8}$O | $^{14}_{7}$N (ex) | $^{14}_{6}$C | $10^3$ yrs |
| $^{18}_{10}$Ne | $^{18}_{9}$F (ex) | $^{18}_{8}$O | |
| $^{22}_{12}$Mg | $^{22}_{11}$Na (ex) | $^{22}_{10}$Ne | |
| $^{26}_{14}$Si | $^{26m}_{13}$Al | $^{26}_{12}$Mg | |
| $^{30}_{16}$S | $^{30}_{15}$P (ex) | $^{30}_{14}$Si | |
| $^{34}_{18}$Ar | $^{34}_{17}$Cl | $^{34}_{16}$S | |
| $^{38}_{20}$Ca | $^{38m}_{19}$K | $^{38}_{18}$Ar | stable |
| $^{42}_{22}$Ti | $^{42}_{21}$Sc | $^{42}_{20}$Ca | |
| $^{46}_{24}$Cr | $^{46}_{23}$V | $^{46}_{22}$Ti | |
| $^{50}_{26}$Fe | $^{50}_{25}$Mn | $^{50}_{24}$Cr | |
| $^{54}_{28}$Ni | $^{54}_{27}$Co | $^{54}_{26}$Fe | |

| Tz = -1 (Parent) | 0 (daughter) | +1 | |
|---|---|---|---|
| $^{26}_{14}$Si | $^{26m}_{13}$Al | $^{26}_{12}$Mg | |
| $^{34}_{18}$Ar | $^{34}_{17}$Cl | $^{34}_{16}$S | |
| $^{38}_{20}$Ca | $^{38m}_{19}$K | $^{38}_{18}$Ar | |
| $^{42}_{22}$Ti | $^{42}_{21}$Sc | $^{42}_{20}$Ca | stable |
| $^{46}_{24}$Cr | $^{46}_{23}$V | $^{46}_{22}$Ti | |
| $^{50}_{26}$Fe | $^{50}_{25}$Mn | $^{50}_{24}$Cr | |
| $^{54}_{28}$Ni | $^{54}_{27}$Co | $^{54}_{26}$Fe | |
| $^{62}_{32}$Ge | $^{62}_{31}$Ga | $^{62}_{30}$Zn | 9 hrs |
| $^{66}_{34}$Se | $^{66}_{33}$As | $^{66}_{32}$Ge | 2 hrs |
| $^{70}_{36}$Kr | $^{70}_{35}$Br | $^{70}_{34}$Se | 41 min |
| $^{74}_{38}$Sr | $^{74}_{37}$Rb | $^{74}_{36}$Kr | 11 min |

However, at N≠Z, disentangling the **ISB** and **symmetry energy** contribution to the neutron skin is non-trivial

# Probing isospin mixing effects from electroweak radii

Strategy: Measure different radii across the isotriplet!

"Isovector monopole operator": $$\vec{M}^{(1)} = \sum_{i=1}^{A} r_i^2 \vec{\hat{T}}(i)$$

Let's consider $(Tz)_i = 0$, $(Tz)_f = +1$.

## Measurement (1): t-dependence in beta decay

Beta decay form factors:

$$\langle f(p_f)|J_W^{\lambda\dagger}(0)|i(p_i)\rangle = f_+(t)(p_i + p_f)^\lambda + f_-(t)(p_i - p_f)^\lambda$$

Recoil effects probe the t-dependence, give the off-diagonal matrix element of the isovector monopole operator:

$$\bar{f}_+(t) = 1 - \frac{t}{6}\frac{\langle f|M_{+1}^{(1)}|i\rangle}{\sqrt{2}M_F} + \mathcal{O}(t^2)$$



Existing recoil expt: TRIUMF, ISOLDE (CERN) etc. Future expt at FRIB?

# Probing isospin mixing effects from electroweak radii

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"Isovector monopole operator": $\vec{M}^{(1)} = \sum_{i=1}^{A} r_i^2 \vec{\hat{T}}(i)$

Let's consider $(Tz)_i = 0$, $(Tz)_f = +1$.

## Measurement (2): p/n distribution radius at $(Tz)_f$=+1

For stable daughter nucleus, fixed-target scattering can be performed to measure $R_p$ and $R_n$ respectively (deduced from charge and weak radii)

Can combine to get another matrix element of the isovector monopole operator:

$$\langle f|M_0^{(1)}|f\rangle = \langle f|\sum_{i=1}^{A} r_i^2 \hat{T}_z(i)|f\rangle = \frac{N}{2}R_{n,f}^2 - \frac{Z}{2}R_{p,f}^2$$



PREX, CREX (JLab), P2, MREX (Mainz)…

# Probing isospin mixing effects from electroweak radii

Strategy: Measure different radii across the isotriplet!

"Isovector monopole operator": $$\vec{M}^{(1)} = \sum_{i=1}^{A} r_i^2 \vec{\hat{T}}(i)$$

Let's consider $(Tz)_i = 0$, $(Tz)_f = +1$.

## Combined experimental observable

If isospin symmetry is exact, the two matrix elements are equal and opposite:

$$\langle f_0 | M_{+1}^{(1)} | i_0 \rangle = -\langle f_0 | M_0^{(1)} | f_0 \rangle$$

Therefore, the combined experimental observable:

$$\Delta M_A^{(1)} \equiv \langle f | M_{+1}^{(1)} | i \rangle + \langle f | M_0^{(1)} | f \rangle$$

provides a clean probe of ISB. **Deviation from zero signifies isospin mixing**

# Probing isospin mixing effects from electroweak radii

Strategy: Measure different radii across the isotriplet!

"Isovector monopole operator":

$$\vec{M}^{(1)} = \sum_{i=1}^{A} r_i^2 \vec{\hat{T}}(i)$$

Let's consider $(Tz)_i = 0$, $(Tz)_f = +1$.

## Measurement (3): Charge radii across the isotriplet

**Nuclear charge radii** are measurable for both stable and unstable nuclei (through atomic spectroscopy)

Assuming $R_{ch} \approx R_p$, the following observable is also a clean probe of ISB:

$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2\right) - Z_0 R_{p,0}^2$$



Possible future measurements: BECOLA at FRIB

# Connection to the ISB correction to $M_F$

Leading source of isospin mixing:

Isovector Coulomb potential in a uniformly charged sphere

$$V_C^{(1)} = \frac{Ze^2}{8\pi R_C^3}\sum_i r_i^2 \hat{T}_z(i) + \ldots$$

$$= \frac{Ze^2}{8\pi R_C^3} M_0^{(1)} + \ldots$$

*Damgaard, 1969 Nucl.Phys.A; Miller and Schwenk, 2008 PRC; Auerbach, 2009 PRC*

$$\Delta M_A^{(1)} \approx -\frac{8\pi R_C^3}{Ze^2}\left\{\frac{1}{3}\sum_a \frac{|\langle a;0||V_C^{(1)}||g;1\rangle|^2}{E_{a,0}-E_{g,1}} + \frac{1}{2}\sum_{a\neq g}\frac{|\langle a;1||V_C^{(1)}||g;1\rangle|^2}{E_{a,1}-E_{g,1}} + \frac{7}{6}\sum_a \frac{|\langle a;2||V_C^{(1)}||g;1\rangle|^2}{E_{a,2}-E_{g,1}}\right\}$$

$$\Delta M_B^{(1)} \approx -\frac{8\pi R_C^3}{Ze^2}\left\{\frac{2}{3}\sum_a \frac{|\langle a;0||V_C^{(1)}||g;1\rangle|^2}{E_{a,0}-E_{g,1}} - \sum_{a\neq g}\frac{|\langle a;1||V_C^{(1)}||g;1\rangle|^2}{E_{a,1}-E_{g,1}} + \frac{1}{3}\sum_a \frac{|\langle a;2||V_C^{(1)}||g;1\rangle|^2}{E_{a,2}-E_{g,1}}\right\}$$

$$\delta_C \approx \frac{1}{3}\sum_a \frac{|\langle a;0||V_C^{(1)}||g;1\rangle|^2}{(E_{a,0}-E_{g,1})^2} + \frac{1}{2}\sum_{a\neq g}\frac{|\langle a;1||V_C^{(1)}||g;1\rangle|^2}{(E_{a,1}-E_{g,1})^2} - \frac{5}{6}\sum_a \frac{|\langle a;2||V_C^{(1)}||g;1\rangle|^2}{(E_{a,2}-E_{g,1})^2}$$

They share **identical reduced matrix elements** in the **T=0,1,2** channels!

Benefits to theorists: Methodologies capable to compute $\delta_C$ can also compute $\Delta M^{(1)}$; the latter can be directly compared to experiment!

# Connection to the ISB correction to $M_F$

Further modeling invoking **isovector monopole dominance**:

*Auerbach, 1983 Phys.Rept*

$$\sum_a \frac{|\langle a;T||V_C^{(1)}||g;1\rangle|^2}{(E_{a,T}-E_{g,1})^n} \to \frac{|\langle M;T||V_C^{(1)}||g;1\rangle|^2}{(E_{M,T}-E_{g,1})^n}$$

Assuming degenerate reduced matrix elements: $\langle M;T||V_C^{(1)}||g;1\rangle \equiv u$ (correction ~ |N-Z|/A)

Energy splitting: $E_{M,T}-E_{g,1}=\xi\omega[1+(T^2+T-4)\kappa/2]$

Reduced matrix elements drop out, leading to a **direct proportionality**:

$$\begin{aligned}\delta_C &\approx -\frac{Ze^2}{8\pi R_C^3}\frac{\kappa(4-13\kappa+12\kappa^2-\kappa^3)}{(\kappa^2-4\kappa+2)(1-2\kappa)(1-\kappa^2)}\frac{1}{\xi\omega}\Delta M_A^{(1)}\\ &\approx -\frac{Ze^2}{8\pi R_C^3}\frac{(4-13\kappa+12\kappa^2-\kappa^3)}{2\kappa(1-2\kappa)(1-\kappa^2)}\frac{1}{\xi\omega}\Delta M_B^{(1)}\end{aligned}$$

Proportionality constants bearing residual model dependence

# Targeted Experimental Precision

**How precise should experiments be** to start probing isospin mixing effects?

The **simple proportionality relation** + **$\delta_c$ available on market** provide useful guidance on the targeted experimental precision!

| Transitions | $\delta_C$ (%) | | | | | $\Delta M_A^{(1)}$ ($\mathrm{fm}^2$) | | | | | $\frac{\Delta M_A^{(1)}}{AR^2/4}$ (%) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | WS | DFT | HF | RPA | Micro | WS | DFT | HF | RPA | Micro | WS | DFT | HF | RPA | Micro |
| $^{26m}\mathrm{Al} \rightarrow {}^{26}\mathrm{Mg}$ | 0.310 | 0.329 | 0.30 | 0.139 | 0.08 | -2.2 | -2.3 | -2.1 | -1.0 | -0.6 | 3.2 | 3.3 | 3.0 | 1.4 | 0.8 |
| $^{34}\mathrm{Cl} \rightarrow {}^{34}\mathrm{S}$ | 0.613 | 0.75 | 0.57 | 0.234 | 0.13 | -5.0 | -6.1 | -4.6 | -1.9 | -1.0 | 4.6 | 5.6 | 4.3 | 1.8 | 1.0 |
| $^{38m}\mathrm{K} \rightarrow {}^{38}\mathrm{Ar}$ | 0.628 | 1.7 | 0.59 | 0.278 | 0.15 | -5.4 | -14.6 | -5.1 | -2.4 | -1.3 | 4.2 | 11.2 | 3.9 | 1.8 | 1.0 |
| $^{42}\mathrm{Sc} \rightarrow {}^{42}\mathrm{Ca}$ | 0.690 | 0.77 | 0.42 | 0.333 | 0.18 | -6.2 | -6.9 | -3.8 | -3.0 | -1.6 | 4.0 | 4.5 | 2.5 | 2.0 | 1.1 |
| $^{46}\mathrm{V} \rightarrow {}^{46}\mathrm{Ti}$ | 0.620 | 0.563 | 0.38 | / | 0.21 | -5.8 | -5.3 | -3.6 | / | -2.0 | 3.3 | 3.0 | 2.0 | / | 1.1 |
| $^{50}\mathrm{Mn} \rightarrow {}^{50}\mathrm{Cr}$ | 0.660 | 0.476 | 0.35 | / | 0.24 | -6.4 | -4.6 | -3.4 | / | -2.4 | 3.1 | 2.3 | 1.7 | / | 1.2 |
| $^{54}\mathrm{Co} \rightarrow {}^{54}\mathrm{Fe}$ | 0.770 | 0.586 | 0.44 | 0.319 | 0.28 | -7.8 | -5.9 | -4.4 | -3.2 | -2.8 | 3.3 | 2.5 | 1.9 | 1.4 | 1.2 |

$\delta_c$ from different models

Expected size of $\Delta M_A^{(1)}$

Targeted relative precision of $R_p^2$, $R_n^2$ measurement

$\Delta M_A^{(1)}$ sensitivity **enhanced by $1/\kappa$**

# Targeted Experimental Precision

**How precise should experiments be** to start probing isospin mixing effects?

The **simple proportionality relation** + $\delta_c$ **available on market** provide useful guidance on the targeted experimental precision!

| Transitions | $\Delta M_B^{(1)}$ (fm$^2$) | | | | | $\frac{\Delta M_B^{(1)}}{AR^2/2}$ (%) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | WS | DFT | HF | RPA | Micro | WS | DFT | HF | RPA | Micro |
| $^{26m}$Al $\to$ $^{26}$Mg | -0.12 | -0.12 | -0.11 | -0.05 | -0.03 | 0.08 | 0.09 | 0.08 | 0.04 | 0.02 |
| $^{34}$Cl $\to$ $^{34}$S | -0.17 | -0.21 | -0.16 | -0.06 | -0.04 | 0.08 | 0.10 | 0.07 | 0.03 | 0.02 |
| $^{38m}$K $\to$ $^{38}$Ar | -0.15 | -0.42 | -0.15 | -0.07 | -0.04 | 0.06 | 0.16 | 0.06 | 0.03 | 0.01 |
| $^{42}$Sc $\to$ $^{42}$Ca | -0.15 | -0.17 | -0.09 | -0.07 | -0.04 | 0.05 | 0.06 | 0.03 | 0.02 | 0.01 |
| $^{46}$V $\to$ $^{46}$Ti | -0.12 | -0.11 | -0.08 | / | -0.04 | 0.03 | 0.03 | 0.02 | / | 0.01 |
| $^{50}$Mn $\to$ $^{50}$Cr | -0.12 | -0.09 | -0.06 | / | -0.04 | 0.03 | 0.02 | 0.02 | / | 0.01 |
| $^{54}$Co $\to$ $^{54}$Fe | -0.13 | -0.10 | -0.07 | -0.05 | -0.05 | 0.03 | 0.02 | 0.02 | 0.01 | 0.01 |

Expected size of $\Delta M_B^{(1)}$

Targeted relative precision of $R_p^2$ measurement



$\Delta M_B^{(1)}$ sensitivity **suppressed by $\kappa$** , but partial results already exists

# Anticipated Synergies

Differential superallowed decay rate (FRIB, ...)

$\Delta M^{(1)}_{A,B}$

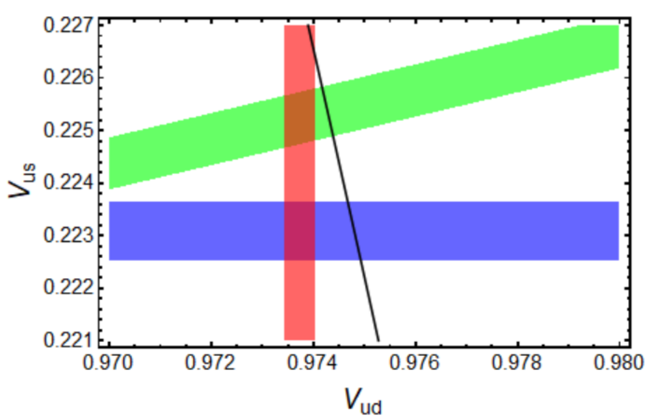
EoS, nuclear astrophysics

Scrutinize the theory relation (nuclear models, ab-initio, ...)

Charge and weak radii measurement (JLab, Mainz, FRIB, ...)

$\delta_C$

$V_{ud}$

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Precision tests of SM; Search for BSM physics

# Summary

- Isospin breaking correction $\delta_C$ is an important SM theory input for the test of first-row CKM unitarity. Current determination suffers from theoretical inconsistencies, model-dependence, and is short of direct experimental constraint.
- We propose a new set of experimental observables $\Delta M_A^{(1)}$ , $\Delta M_B^{(1)}$ from the measurement of electroweak nuclear radii across the isotriplet, as a clean probe of isospin mixing effects.
- We show that $\Delta M_A^{(1)}$, $\Delta M_B^{(1)}$ probe the same physics as $\delta_C$ , thus serves as a strong constraint to the latter. They also provide useful consistency checks to theory calculations.
- Existing models indicate that measurements of $R_p^2$, $R_n^2$ to ~1% may start to probe isospin mixing effects.
- The new idea may motivate synergies between theory and experimental communities in physics of rare isotopes, electron scattering and nuclear astrophysics.